Some Constructions of Pseudo-Collarable 1-Ended Manifolds

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- In either case, the map f is a $\mathbb{Z}Q$-homology isomorphism

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Theorem (R., 2014)

Let M be a manifold of dimension 6 or higher, with $\pi_1(M) \cong Q$. Let K be a finitely presented superperfect group. Let G be a semi-direct product of Q by K, $G = K \rtimes Q$. Then there is a cobordism (W, M, M_-) with $\pi_1(M_-) \cong G$ and $M \hookrightarrow W$ a simple homotopy equivalence

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- (Semi-direct products are the simplest kind of group extensions; direct products are one example)

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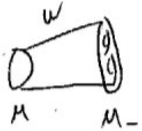

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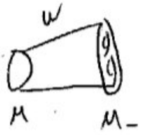

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- Note (W, M_-, M) (read upside-down, with the roles of M and M_- reversed) is a plus cobordism (so $(M_-)^+ \approx M$)

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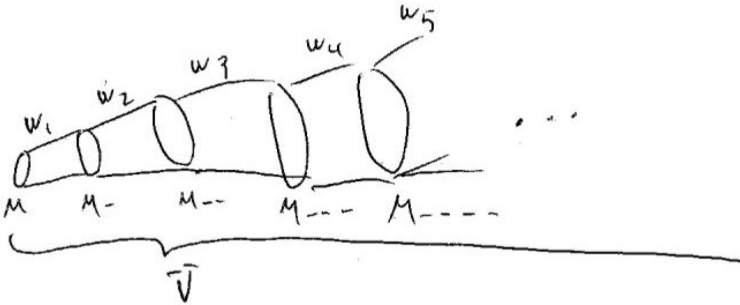
- 

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- This is one advantage of using semi-direct products over direct products

Uncountable Many Pseudo-Collars

- A finitely presented group P is called <u>allowable</u> if it contains a countably infinite group $A = \{a_n \mid n \in \mathbb{N}\}$ with the property that if $\phi : K \to K$ is an isomorphism and $\phi(a_i) = a_j^{\pm 1}$, then $i = j$

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Theorem (R., 2016)

Let M be a manifold of dimension $n \geq 6$ with fundamental group $\mathbb{Z}$. Let P be a finitely presented, allowable, superperfect, centerless, Hopfian group, and let $S = P * P$. Then there are uncountably many $(n+1)$-dimensional, pseudo-collarable, 1-ended manifolds V with boundary M

- V will break up into semi-s-cobordisms (W_j, M_{j-1}, M_j), where $G_j \cong S \rtimes G_{j-1}$, $G_j = \pi_1(M_j)$

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- We will produce one V for each $\omega \in \Pi_{i=1}^{\infty}\{0, 1\}$

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- The hard part is proving that the resulting pro-fundamental group systems at infinity are all non-isomorphic
- For example, if $Q = \Pi_{i=1}^{\infty}\mathbb{Z}$, $K_1 = \mathbb{Z}$, and $K_2 = \mathbb{Z} \times \mathbb{Z}$, then $G_1 = K_1 \times Q$ and $G_2 = K_2 \times Q$ are isomorphic, even though $K_1 \ncong K_2$

Sketch of Proof

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Lemma

Let A, B, C, and D be nontrivial groups and ley $\phi : A \times B \to C * D$ be a epimorphism. Then either $\phi(A \times \{1\})$ is all of $C * D$ and $\phi(\{1\} \times B)$ is trivial or $\phi(A \times \{1\})$ is trivial and $\phi(\{1\} \times B)$ is all of $C * D$

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- The proof uses the fact that a free product is never an internal direct product

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Lemma (The Straightening-Up Lemma ($n = m$))

Let $n = m$, let S be a free product, and let $\psi : S \times S \times \ldots \times S$ (n copies) $\to S \times S \times \ldots \times S$ (m copies) be an isomorphism. Write $\psi_{i,j}$ for $\pi_{S_j} \circ \psi|_{S_i}$. Then ψ splits as n isomorphisms $\psi_{i,\sigma(i)}$, with σ a permutation, with all other $\psi_{i,j}$'s being the trivial map

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Lemma (The Straightening-Up Corollary ($n > m$))

Let $n > m$, let S be a Hopfian free product, and let $\psi : S \times S \times \ldots \times S$ (n copies) $\to S \times S \times \ldots \times S$ (m copies) be an epimorphism. Write $\psi_{i,j}$ for $\pi_{S_j} \circ \psi|_{S_i}$. Then ψ splits as m isomorphisms $\psi_{\sigma^{-1}(i),i}$, with σ a permutation, with all other $\psi_{i,j}$'s being the trivial map

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- (This particular kind of outer action is called a partial conjugation)

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- Already, due to a suggestion by Jason Manning, we have a countable collection of fundamental groups of hyperbolic 3-manifolds that are allowable and meet all the requirements for a kernel group, for each of which we produce an uncountable collection of pseudo-collars

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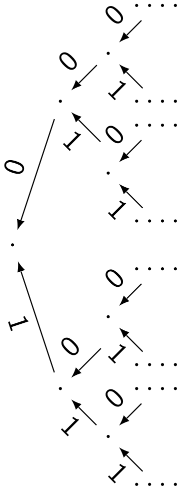

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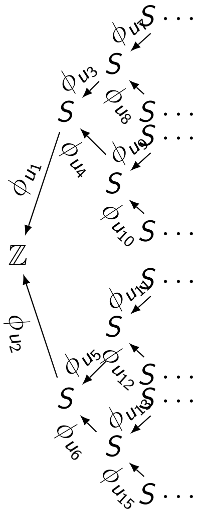

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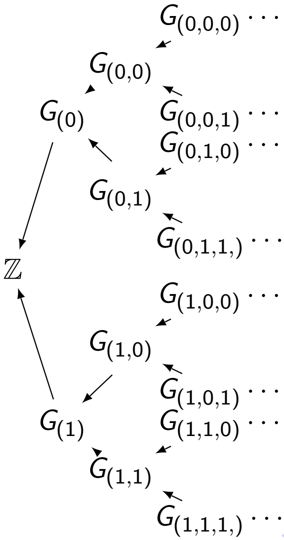

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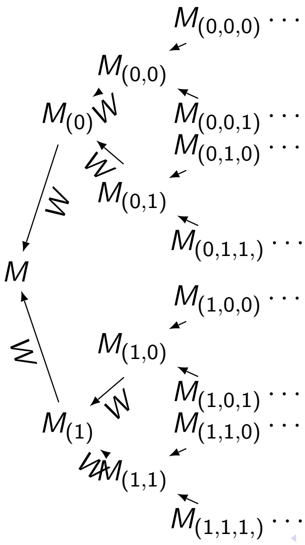

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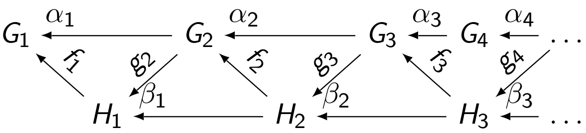
- Now, for each $\omega \in \Omega$, let (ω, n) be the corresponding finite sequence of length n
- Corresponding to each $\omega \in \Omega$, we have an inverse sequence of groups $(G_{(\omega,n)}, \alpha_{(\omega,n)})$
- Two inverse sequences are pro-isomorphic if and only if, after passing to subsequences, they may be put into a ladder diagram



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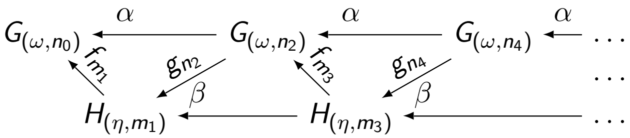
- Suppose, after passing to subsequences, we have their pro-fundamental group systems at infinity fitting into a ladder diagram



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- But, ω and η only agree up to n_0 and cannot have ϕ_{u_i}'s with the same u_i's on the remaining $m_1 - n_0$ corresponding copies of S!

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Lemma

By passing to a further subsequence if necessary, we may assume $n_0 < m_1 < n_2 < m_3 < \dots$

- Now, g_{n_2} fits the form for the Conder Isomorphism Corollary $(n > m)$, so it must be onto m_1 copies of S and corresponding copies of S must have ϕ_{u_i}'s with the same u_i's in corresponding copies of the S's
- But, ω and η only agree up to n_0 and cannot have ϕ_{u_i}'s with the same u_i's on the remaining $m_1 - n_0$ corresponding copies of S!
- This concludes the proof

The End

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